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# CALCULUS.

215. Proposed by PROFESSOR B. F. FINKEL, A. M., 4038 Locust Street, Philadelphia, Pa.

Prove that, if the differential equation  $cydx - (y + a + bx)dy - nx(xdy - ydx) = 0$ , be transformed into an equation between  $u$  and  $x$  by the substitution  $u(y + a + bx + nx^2) = y(c + nx)$ , then the variables are separable; and reduce the equation to the form  $dv/\phi(v) = dx/\phi(x)$  by the further substitution  $v = au + \beta$ ,  $a$  and  $\beta$  being suitably determined. *Euler*. [Forsyth's *Differential Equations*, p. 48, Ex. 4.]

Solution by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

The first equation may be written

$$\frac{dy}{dx} = \frac{(c + nx)y}{y + a + bx + nx^2}. \quad \text{Thus } \frac{dy}{dx} = u,$$

and as  $u(y + a + bx + nx^2) = (c + nx)y$ , we have by differentiating with respect to  $x$ , writing  $u$  for  $\frac{dy}{dx}$ , and  $\frac{u(a + bx + nx^2)}{c + nx - u}$  for  $y$ , and re-arranging,

$$\frac{du}{[c^2 - bc + na + u(b - 2c) + u^2]} = \frac{dx}{(a + bx + nx^2)(c + nx)}.$$

This is of the form  $\frac{du}{f(u)} = \frac{dx}{\phi(x)}$ .

Let  $u = c + nv$ , then  $du = ndv$ ,  $f(u) = n(a + bv + nv^2)(c + nv)$ . Hence,

$$\frac{dv}{(a + bv + nv^2)(c + nv)} = \frac{dx}{(a + bx + nx^2)(c + nx)}$$

which is of the form  $\frac{dv}{\phi(v)} = \frac{dx}{\phi(x)}$ .

Also solved by W. W. Landis, and G. B. M. Zerr.

## DIOPHANTINE ANALYSIS.

132. Proposed by O. E. GLENN, Ph. D., Springfield, Mo.

Disregarding the order of  $\lambda$ ,  $\mu$ ,  $\nu$ , how many sets of solutions has the congruence  $\lambda + \mu + \nu \equiv 0 \pmod{p-1}$  ( $p$  prime)? [A. E. Western.]

\*Solution by the PROPOSER.

Let  $n_i$  be the number of solutions in which  $i$  of the numbers  $\lambda$ ,  $\mu$ ,  $\nu$  are equal. If  $p \equiv 1 \pmod{3}$   $n_3 = 3$ , the solutions being

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\*See problems for solution, *Diophantine Analysis*, No. 134.

$$\lambda \equiv \mu \equiv \nu \equiv 0, \text{ or } \frac{p-1}{3} \text{ or } \frac{2(p-1)}{3} \pmod{p-1}.$$

If  $p \equiv 2 \pmod{3}$ ,  $n_3 = 1$ , viz.,  $\lambda \equiv \mu \equiv \nu \equiv 0 \pmod{p-1}$ . Next when two are equal so that the congruence is  $\lambda + 2\mu \equiv 0 \pmod{p-1}$ ,  $\mu$  cannot be  $\equiv 0, \frac{p-1}{3}$ , or  $\frac{2(p-1)}{3}$  for then it would be  $\equiv \lambda$ . With these exceptions  $\mu$  can have any value, and for each value of  $\mu$  the congruence gives one value of  $\lambda$ . Hence if  $p \equiv 1 \pmod{3}$ ,  $n_2 = p-4$ , and when  $p \equiv 2 \pmod{3}$ ,  $n_2 = p-2$ .

If account be taken of the order of  $\lambda, \mu, \nu$  the total number of solutions of the congruence is  $(p-1)^2$  since each unknown may assume  $p-1$  values and when two are fixed the other is determined by the congruence. In terms of  $n_1, n_2$  and  $n_3$  the above totality of solutions equals  $6n_1 + 3n_2 + n_3$ . Hence  $6n_1 + 3n_2 + n_3 = (p-1)^2$ , so that if  $p \equiv 1 \pmod{3}$ ,  $n_1 = \frac{1}{6}(p^2 - 5p + 10)$ , and when  $p \equiv 2 \pmod{3}$ ,  $n_1 = \frac{1}{6}(p^2 - 5p + 6)$ . Finally, when  $p \equiv 1 \pmod{3}$ , the total number ( $N$ ) of solutions, disregarding order, of the given congruence is

$$N = 3 + p - 4 + \frac{1}{6}(p^2 - 5p + 10) = \frac{1}{6}(p^2 + p + 4),$$

and if  $p \equiv 2 \pmod{3}$ ,

$$N = 1 + p - 2 + \frac{1}{6}(p^2 - 5p + 6) = \frac{1}{6}(p^2 + p).$$

## GEOMETRY.

278. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

$AF, MN$  are parallel lines indefinitely extended toward  $FN$ ; at right angles to  $AF, MN$  is  $AM$  of length 22; upon the base  $AB$ , which is in line with  $AM$ , is the triangle  $ABC$  whose sides are  $AB=21, BC=10, AC=17$ ; find the sides of the maximum similar triangle with base extending from  $B$  to some point in  $AF$ , the vertex in line with  $MN$ .

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let  $D$  be the vertex in  $AF$ ,  $E$  the vertex in  $MN$ ,  $AD=x, ME=y$ . Two cases are possible,  $BD$  the short side or  $BD$  the long side. The latter gives the maximum. Let  $BD=21M, DE=10M, BE=17M$ . Then

$$441M^2 = 441 + x^2 \dots\dots\dots(1),$$

$$289M^2 = 1 + y^2 \dots\dots\dots(2),$$

$$100M^2 = (x-y)^2 + 484 \dots\dots\dots(3).$$

Substitution of  $x, y$  from (1) and (2) in (3) gives  $\sqrt{[(M^2-1)(289M^2-1)]} = 15M^2 + 1$ . Hence  $M^2=5, M=\sqrt{5}$ , and the sides are  $21\sqrt{5}, 17\sqrt{5}, 10\sqrt{5}$ .

Also solved by the Proposer.